Graph Signal Processing for Dynamic Geometry

PHILIP A. CHOU,
HA Q. NGUYEN, MINH DO,
DORINA THANOU, PASCAL FROSSARD

WORKSHOP ON GRAPH SIGNAL PROCESSING FOR IMAGING
OCTOBER 31, 2014
Outline

Graph Signal Processing for Static Geometry

Graph Signal Processing for Dynamic Geometry (topology consistent over time)

Graph Signal Processing for Dynamic Geometry (topology not consistent over time)
Eigenvectors of a Mesh
Spectrum of Geometry as a Signal
Smoothing by Ideal Low-Pass Filtering
Dealing with Complexity

Traditional images: process in blocks or wavelets

Arbitrary meshes: not so easy
Multiresolution Graph

Coarsest graph
Multiresolution Graph
Multiresolution Graph
Multiresolution Graph
Multiresolution Graph

Finest graph
Catmull-Clark Subdivision

Lowpass subband

Highpass subband
Catmull-Clark Subdivision
Catmull-Clark Subdivision
Catmull-Clark Subdivision
Catmull-Clark Subdivision

- Lowpass subband
- Highpass subband
Graph Wavelet Filter Banks (GWFBs)

Critical sampled, compactly supported, near-orthogonal, designed in spectral domain

Building block: two-channel filter bank on a bipartite graph

[Narang & Ortega 2013]
Outline

Graph Signal Processing for Static Geometry

Graph Signal Processing for Dynamic Geometry (topology consistent over time)

Graph Signal Processing for Dynamic Geometry (topology not consistent over time)
Dynamic geometry compression competition dataset\(^1\)

Experiments carried over five sequences: Handstand, Dance, Dog, Wheel, Skirt

Performance measured in terms of Metro distance (RMS)

\[
d_{RMS}(X, Y) = \max\{\text{rms } \inf_{x \in X} d(x, y), \text{rms } \inf_{y \in Y} d(x, y)\}
\]

\(^1\) [http://www.geometrycompression.org/](http://www.geometrycompression.org/)
Trellis for Temporally Consistent Graphs
Predictive Transform Coding
Rate-Distortion Performance
Outline

Graph Signal Processing for Static Geometry

Graph Signal Processing for Dynamic Geometry (topology consistent over time)

Graph Signal Processing for Dynamic Geometry (topology not consistent over time)
Motion Estimation and Compensation

From graphs $G_t$ and $G_{t+1}$:

1. Extract feature vectors in each graph, $f_m (m \in G_t)$ and $f_n (n \in G_{t+1})$
2. Compute feature (dis)similarity, $score(f_m, f_n)$
3. Find best matching points, $m = n^*$ for each $n$
4. Keep matching pairs of interest, $(n_i^*, n_i)$
5. Compute sparse set of motion vectors, $mv_{n_i} = p_{n_i} - p_{n_i^*}$
6. Smooth to get dense motion field, $\bar{mv}_{n}$ for all $n \in G_{t+1}$
7. Warp graph $G_{t+1}$ and connect to graph $G_t$
8. Interpolate signals on $G_{t+1}$ from signals on $G_t$
Features: Spectral Signatures on Graphs

Based on spectral graph wavelets (SGWT)*: dilated, translated versions of a bandpass kernel designed in the graph spectral domain of the graph Laplacian

\[ \psi_{t,n}(m) = \sum_{l=0}^{N-1} g(t\lambda_l)\chi^*_l(n)\chi_l(m) \]

Spectral Graph Wavelets $\psi_{t,n}$ (example)

Can be efficiently implemented by approximating the wavelet operator with powers of the Laplacian (Chebyshev polynomials)
Feature Extraction

For each node $n \in \mathcal{G}$, define the octant indicator functions:

1. $o_{1,n}(k) = 1_{\{x(k) \geq x(n), y(k) \geq y(n), z(k) \geq z(n)\}}(k), \ k \in \mathcal{G}$
2. $o_{1,n}(k) = 1_{\{x(k) \geq x(n), y(k) \geq y(n), z(k) < z(n)\}}(k), \ k \in \mathcal{G}$

... 
8. $o_{1,n}(k) = 1_{\{x(k) < x(n), y(k) < y(n), z(k) < z(n)\}}(k), \ k \in \mathcal{G}$

For each color and geometry component $s \in \{r, g, b, x, y, z\}$ at that node compute the wavelet coefficients:

$$f_{n,t,o_i,s} = \langle s \cdot o_{i,n}, \psi_{t,n} \rangle \text{ for } i = 1, \ldots, 8 \text{ and } t = t_1, \ldots, t_{\text{max}}$$

Feature vector is concatenation of these wavelet coefficients: $f_n = \{f_{n,t,o_i,s}\}$
Feature (dis)similarity and matching

For all $m \in \mathcal{G}_t$ and $n \in \mathcal{G}_{t+1}$ compute the Mahalanobis distance

$$score(m, n) = (f_m - f_n)^T P (f_m - f_n)$$

where $P$ is a covariance matrix estimated (i.e., trained) from features known to be in correspondence.

For each $n \in \mathcal{G}_{t+1}$ define its best match in $\mathcal{G}_t$:

$$n^* = \arg \min_{m \in \mathcal{G}_t} score(m, n)$$

$$bestscore(n) = \min_{m \in \mathcal{G}_t} score(m, n)$$

Keep sparse set of matching points $(n_i^*, n_i)$ s.t. each region in $\mathcal{G}_{t+1}$ has at least one point $n_i$ and $bestscore(n_i) \leq \text{thresh}$. 
Sparse set of motion vectors

Compute motion vectors for the sparse set of matching points:

\[ m v_n^* = p_n - p_n^* \]

where \( p_n = [x_n, y_n, z_n]^T \) is the position of vertex \( n \)

Approximate \( \text{score}(m, n) \) for \( m \) near \( n^* \) in terms of \( m v_n = p_n - p_m \):

\[ \text{score}(m, n) \approx \text{bestscore}(n) + (p_n - p_m)^T M_n^{-1} (p_n - p_m) \]

where \( M_n = \frac{1}{|\mathcal{N}_{n^*}^2|} \sum_{m \in \mathcal{N}_{n^*}^2} \frac{(p_m - p_n^*)^T (p_m - p_n^*)}{\text{score}(m,n) - \text{bestscore}(n)} \) and \( \mathcal{N}_{n^*}^2 \) is the two-hop neighborhood of \( n^* \) in \( G_t \)
Smooth dense set of motion vectors

Letting $Q = \begin{bmatrix} M_1^{-1} & \cdots & 0_{3 \times 3} \\ \vdots & \ddots & \vdots \\ 0_{3 \times 3} & \cdots & M_N^{-1} \end{bmatrix}$ where $M_n^{-1} = 0_{3 \times 3}$ if $n \notin$ sparse set,

smooth the signal of motion vectors:

\[
\tilde{m}v^* = \arg \min (mv - mv^*)^T Q (mv - mv^*) + \lambda \sum_{i=1}^{3} (S_i mv)^T L_t (S_i mv)
\]

where $L_t$ is the graph Laplacian and $S_i$ is a selection matrix

Penalize excess matching score on the sparse set
Impose smoothness of the motion vectors on the graph
Closed form solution

This has a closed form solution,

$$\tilde{m}v^* = \left( Q + \lambda \sum_{i=1}^{3} S_i^T L_t S_i \right)^{-1} Qm v^*$$

which can be solved iteratively using MINRES-QLP for efficiency on large systems

Graph $G_t$ warped to $G_{t+1}$ – Example 1
Graph $G_t$ warped to $G_{t+1}$ – Example 2
Graph $G_t$ warped to $G_{t+1}$ – Example 3
Color Prediction – Example 1

\[ SNR = 10 \log_{10} \left( \frac{\| \text{current\_color} \|^2}{\| \text{prediction\_error} \|^2} \right) \]
Color Prediction – Example 2

Prediction from the previous frame
Prediction from motion compensated previous frame

SNR (dB)

Octree stepsizes
Conclusion

Geometry is a natural application of Graph Signal Processing

Dynamic Geometry, especially where topology is temporally inconsistent, is fertile ground for new problems in Graph Signal Processing